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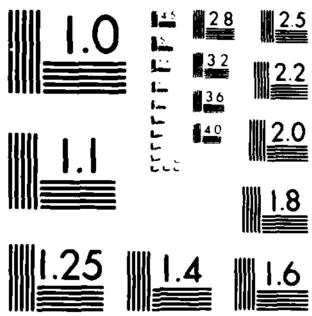

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ON THE USE OF RANK TESTS AND ESTIMATES IN THE LINEAR MODEL

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ABSTRACT

The current status, including small-sample behavior and ease of computation, of rank-based estimates and tests in the general linear model is reviewed. For the important special case of Wilcoxon scores, details of application of various procedures are discussed. The three different testing methods considered may each be motivated by connecting it to one of three forms of the usual least-squares F statistic. Possible algorithms for computation of rank-based estimates and tests are presented. Each procedure is applied to an example using data. Finally, the technical assumptions made to obtain large-sample properties of these procedures, including the general-scores case, are outlined and discussed.

1. INTRODUCTION

The purpose of this paper is to review the current status of statistical methods based on ranks in the linear model. Twenty years ago Hodges and Lehmann (1962) proposed aligned rank tests in a two-way layout. Their approach was to remove the effect of the nuisance parameter and then construct a test of hypothesis on the residuals. Aligned rank tests have received much attention in the literature since 1962. Koul (1970) discussed aligned rank tests for regression with two independent variables, and improvements were suggested by Puri and Sen (1973). Recent papers by Adichie (1978) and Sen and Puri (1977) describe aligned rank tests for the univariate and multivariate linear models, respectively.

Surprisingly, for all the attention accorded to aligned rank tests in the past twenty years, they are not widely available to the data analyst. For example, they are not currently contained in any of the statistical packages, so some degree of special programming is required to implement them.

There are alternative approaches to the aligned rank tests. By substituting for least squares a measure of dispersion based on ranks, due to Jaeckel (1972), McKean and Hettmansperger (1976) proposed a test statistic based on the reduction in dispersion due to fitting the reduced and full models. This method uses rank estimates of the regression parameters. Rank estimates in regression were first proposed by Adichie (1967) as extensions of the Hodges and Lehmann (1963) rank estimates of location. Another approach, not discussed in the literature of nonparametric tests, is to construct a quadratic form based on an R-estimate of the parameters in the linear model.

In considering tests based on ranks we are interested in various practical aspects. We will take as given that the procedures are asymptotically equivalent in the sense of Pitman efficiency and that they have the same asymptotic null distribution. The practical questions are: (1) Does a nominal size-α test maintain its size for small to moderate sample sizes? The answer, based on simulations, usually is not quite, and some tuning of the test statistic generally results. (2) What methods are available for computing the test? Can the computations be carried out using existing packages, or are special programs required? If iterative methods are used in computing the tests what can be said about convergence? and (3) Are there small sample differences in the powers of the various tests?

We will describe three methods of constructing tests based on ranks. We will summarize the state of each method in the light of the three questions above. An example will be given in Section 4 illustrating the three approaches on a data set. Finally, since the legitimacy of these methods is based on asymptotic theory, we have provided a list of assumptions that have been used by various researchers to establish the asymptotic distributions.

It is disappointing to note that little is known about the behavior of rank tests and estimates for small samples. Further, they are not generally easy to compute. They require special programs for their implementation with the exception of an aligned rank test in Section 2i. They are not currently available in any statistical computing packages; however, in 1983 a rank-regression command will be available in the Minitab statistical computing system. The output from this command will contain the rank estimate described in Section 2ii and the rank tests described in Sections 2ii and 2iii.

2. RANK TESTS AND ESTIMATES

Problems in analysis of variance, analysis of covariance and regression can often be treated in a unified manner by casting them in terms of a general linear model. The recent books by Draper and Smith (1981) and Neter and Wasserman (1974) contain many examples.

We begin with a linear model for the nx1 vector y of observations, specified by

$$\begin{aligned} y &= 1\alpha + X\beta + e \\ &= 1\alpha^* + X_c\beta + e \, , \end{aligned} \tag{1}$$

where α is the scalar intercept, β is a px1 vector of regression parameters, X is an nxp design matrix and e is an nx1 vector of random errors. In the second equation, we have the centered design matrix $X_c = X - 1\bar{X}'$ where $\bar{X}'$ is a 1xp vector of column means from X and $\alpha^* = \alpha + \bar{X}'\beta$. The details of an example, with data, are given in Section 4.

Working assumptions will be listed as they are needed in the discussion; a full list of technical assumptions is given in Section 5.

<u>ASSUMPTION</u>: A1. We suppose the n errors are independent, identically distributed according to a continuous distribution which has arbitrary shape and median 0.

<u>ASSUMPTION</u>: A2. Suppose X_c is of full rank p. We partition β into two parts: β_1 is (p-q)x1 and β_2 is qx1. Hence the model (1) can be written

$$y = 1\alpha^* + X_{1c}\beta_1 + X_{2c}\beta_2 + e. \tag{2}$$

We will consider tests of

$$H_o:\beta_2=0 \ ,$$

so that β_1 is a (p-q)x1 vector of nuisance parameters.

Before turning to the rank tests we will present the F statistic in a form that will motivate the introduction of the aligned rank test.

Consider, first, the F statistic for $H_o:\beta=0$. There are no nuisance parameters, and

$$F = \frac{y'X_c(X_c'X_c)^{-1}X_c'y}{p\ \hat{\sigma}^2} \ , \tag{3}$$

where $\hat{\sigma}^2$ is the usual unbiased estimate of the error variance σ^2, assumed to be finite.

The general F statistic for $H_o:\beta_2=0$ can be derived from (3) by removing the effects of the nuisance parameters β_1 from both y and X_c before applying (3). Hence in (3) replace y by $y - X_{1c}\hat{\hat{\beta}}_1$, where $\hat{\hat{\beta}}_1 = (X_{1c}'X_{1c})^{-1}X_{1c}'y$, the reduced model least squares estimate, and replace X_c by $Z = X_{2c} - X_{1c}(X_{1c}'X_{1c})^{-1}X_{1c}'X_{2c}$. Now p is replaced by q, the dimension of β_2 and (3) becomes the usual F statistic, written in unusual form,

$$F = \frac{(y-X_{1c}\hat{\hat{\beta}}_1)'Z(Z'Z)^{-1}Z'(y-X_{1c}\hat{\hat{\beta}}_1)}{p\ \hat{\sigma}^2} \ . \tag{4}$$

Further, a bit of matrix algebra shows that

$$Z(Z'Z)^{-1}Z' = X_c(X_c'X_c)^{-1}X_c' - X_{1c}(X_{1c}'X_{1c})^{-1}X_{1c}' \ . \tag{5}$$

(i) Aligned Rank Tests

These tests are easiest to implement when we use the Wilcoxon score function; other score functions are discussed in Section 5. Let

$$\phi(u) = (12)^{1/2}(u - 1/2) , \tag{6}$$

and define $a(i) = \phi(i/(n+1))$. Then $a(1) \leq \ldots \leq a(n)$ are called the Wilcoxon scores. Note that $\int_o^1 \phi(u)du=0$, $\int_o^1 \phi^2(u)du=1$, and $\sum_1^n a(i)=0$.

We will let $a(R(\beta))$ denote the nx1 vector whose i^{th} component is $a(R(y_i-x_{ic}'\beta))$, where $R(y_i-x_{ic}'\beta)$ is the rank of $y_i-x_{ic}'\beta$ among the n uncentered residuals.

<u>ASSUMPTION</u>: A3. Suppose $\hat{\hat{\beta}}_1$ is a reduced model estimate such that $\hat{\hat{\beta}}_1(y+b) = \hat{\hat{\beta}}_1(y)+b$ and $n^{1/2}(\hat{\hat{\beta}}_1-\beta_1)=0_p(1)$.

If the error distribution has finite variance and if the maximum leverage tends to 0, then the results of Huber (p 157, 1981) show that the least-squares estimate satisfies A3. For testing $H_o:\beta_2=0$ we first align the observations and construct $a(R(\hat{\hat{\beta}}_1))$, the vector of rank-scored reduced-model residuals. The aligned rank test statistic is constructed from the numerator of (4) by replacing the residuals with $a(R(\hat{\hat{\beta}}_1))$:

$$A = a'(R(\hat{\hat{\beta}}_1))Z(Z'Z)^{-1}Z'a(R(\hat{\hat{\beta}}_1)). \tag{7}$$

Provided the technical assumptions in Section 5 are satisfied, Adichie (1978) showed that A is asymptotically chi-squared with q degrees of freedom. Hence a large sample test of $H_o:\beta_2=0$ rejects H_o if $A \geq \chi^2_\alpha(q)$. Note that A is very easy to compute with a computer package that contains both least squares and ranking capibilities. Computation is carried out in the following way:

C1. Find the reduced-model least-squares residuals,

C2. Find the ranks of these residuals and calculate
$(12)^{1/2}\ ((n+1)^{-1}R(y_i - x_{ic}'\hat{\hat{\beta}}_1) - 1/2),\ i=1,\ \ldots\ n,$

C3. Compute the F statistic on the values in C2; then A is the numerator sum of squares.

Note that for each hypothesis of the form $H_o:\beta_2=0$ which is to be tested a new set of reduced model residuals must be computed. Minitab and SAS programs are provided in Section 3.

Summary: We consider the three questions raised in the Introduction. (1) There are no published studies of the small sample properties of A. We do not know if the test, which is not nonparametric for finite sample size, maintains its level near the nominal level. We do not know how large the sample size should be before it is reasonable to use A. There are many possibilities for choice of $\hat{\hat{\beta}}_1$. The least-squares estimate or a rank estimate discussed in the next subsection are possible candidates. We do not know how the choice of $\hat{\hat{\beta}}_1$ effects the level of the test for small samples; asymptotically it makes no difference. In a small, unpublished study, Hettmansperger and McKean (1981) find that in testing for parallelism, the simulated levels of A were more erractic than the levels of the other tests discussed below. Further, there was some indication that the presence of good design points with moderate leverage can make A extremely conservative.

(2) As described above, if the least-squares reduced model estimate is used, A is easy to compute. It would seem more natural to align the observations using the rank estimate in (ii); see Sen and Puri (1977). Rank estimates present computational difficulties that are discussed in Section 3. (3) There are no published studies on small-sample power of A. In the unpublished report by Hettmansperger and McKean (1981), A had very little power when there was a moderate leverage point in the parallelism design.

(ii) Wald Test Statistic

This test is constructed from a quadratic form in the full model rank estimate of β; see Rao (1973) for a general discussion of the Wald test statistic. The computation of the rank estimate requires specific programs that are not generally available in statistical packages. In Section 3 we discuss the computational problems that must be overcome to be able to implement the Wald test. By 1983 there will be a rank-regression command in the Minitab statistical computing system which will provide all of the necessary computations. Until that time special programs are required to implement the test.

We begin with Jaeckel's (1972) measure of dispersion of the residuals. Using (6), define

$$\begin{aligned} D(\beta) &= a'(R(\beta))(y-X_c\beta) \\ &= \sum_{i=1}^{n} a(R(y_i-x_{ic}'\beta))(y_i-x_{ic}'\beta) \end{aligned} \tag{8}$$

and

$$S(\beta) = X_c'a(R(\beta)). \tag{9}$$

The j^{th} component of the px1 vector $S(\beta)$ is given by $S_j(\beta) = \sum_{i=1}^{n} (x_{ij}-\bar{x}_j)a(R(y_i-x_{ic}'\beta))$, the rank test statistic corresponding to the j^{th} component of β. Jaeckel points out that $-S(\beta)$ is essentially the gradient of $D(\beta)$, so setting (9) to zero yields a set of non-linear normal equations derived from $D(\beta)$.

A rank estimate $\hat{\beta}$ minimizes $D(\beta)$ or solves $S(\beta) \doteq 0$. Jureckova (1971) suggests the equivalent method of minimizing $\Sigma|S_j(\beta)|$.

<u>ASSUMPTION</u>: A4. Suppose $n^{-1}(X_c'X_c)$ converges to a positive definite matrix as $n \to \infty$.

Then the limiting distribution of $n^{1/2}(\hat{\beta}-\beta)$ is $MVN(0, \tau^2 n(X_c'X_c)^{-1})$, where

$$\tau^{-2} = 12\{\int_{-\infty}^{\infty} f^2(x)dx\}^2 \tag{10}$$

and $f(x)$ is the density of the error distribution. See Section 5 for a discussion of score functions other than the Wilcoxon score function.

Let $H = [0,I]$ where I is the qxq identity matris; then $H_o:\beta_2=0$ can be written as $\beta_o:H=0$. Now, $n^{1/2}(H\hat{\beta} - H\beta)$ has a limiting covariance matrix $\tau^2 nH(X_c'X_c)^{-1}H'$, and the Wald statistic is

$$W = \frac{(H\hat{\beta})'[H(X_c'X_c)^{-1}H']^{-1}(H\hat{\beta})}{\hat{\tau}^2} . \tag{11}$$

The statistic W is analogous to the corresponding form of the F statistic where $\hat{\beta}$ is the least-squares estimate and τ^2 is replaced by σ^2. Graybill (p 184, 1976) discusses this form of F in detail. Provided the technical assumptions in Section 5 are met W has a limiting chi-squared distribution with q degrees of freedom. Chiang and Puri (1982) also discuss W.

To carry out the test we need a consistent estimate of τ^2. Let $w(x) = 1$ for $|x| \le 1/2$ and 0 otherwise; then the following rectangular window estimate of $\gamma = \int f^2(x)dx$ is consistent:

$$\hat{\gamma} = \frac{1}{n^{3/2}h_n} + \frac{1}{n(n-1)h_n} \sum_{i \ne j}\sum w\left(\frac{r_i - r_j}{h_n}\right), \tag{12}$$

where $r_i = y_i - x_{ic}'\hat{\beta}$ and

$$h_n = (4.1)n^{-1/2}(r_{(.75n)} - r_{(.25n)}) . \tag{13}$$

The window width h_n incorporates a resistant estimate of scale, the interquartile range of the residuals, and a normalizing factor. The factor 4.1, used in (13), corresponds to a normal error distribution. The estimate (12) is a modification of a window estimate in the independent, identically distributed case proposed by Schuster (1974) and independently by Schweder (1975). The extension to the linear model is discussed by Aubuchon (1982).

We now take $\tau^2 = 1/(12\,\gamma^2)$ in (10). Then the test rejects $H_o:\beta_2=0$ if $W \ge \chi^2_\alpha(q)$. This test is illustrated in Section 4.

An alternative estimate of τ is available with the additional assumption of symmetry of the error distribution.

ASSUMPTION: A5. Suppose the underlying error distribution is symmetric about 0.

Let $W_{(1)} \le \cdots \le W_{(N)}$, $N = n(n+1)/2$ be the ordered set of all pairwise averages of the residuals, including the residuals. These averages are referred to as Walsh averages. Let c by the lower critical point of a two-sided size-α Wilcoxon signed-rank test. Then under assumption A5, McKean and Hettmansperger (1976) show that

$$\tau^* = n^{1/2}(W_{(N-c)} - W_{(c+1)})/(2Z_{\alpha/2}) \tag{14}$$

where $2Z_{\alpha/2}$ is the $1-\alpha$ interpercentile range of the standard normal distribution, is a consistent estimate of τ in (10). This extends the results of Lehmann (1963) in the independent and identically distributed case. A discussion for general scores is given in Section 5.

We complete this subsection with a short discussion of k-step rank estimates $\hat{\beta}^{(k)}$, proposed by McKean and Hettmansperger (1978). Let $\beta^{(0)}$ denote the least squares estimate of β and compute $S(\hat{\beta}^{(0)})$ from (9) and $\hat{\tau}^{(0)}$ from (12), (or (14)), then, using a linear approximation to $S(\beta)$ discussed by Jureckova (1971), form

$$\hat{\beta}^{(1)} = \hat{\beta}^{(0)} + \hat{\tau}^{(0)}(X_c'X_c)^{-1}S(\hat{\beta}^{(0)}). \tag{15}$$

The estimate $\hat{\beta}^{(k)}$ formed by iterating (15) has the same asymptotic distribution for any k=1, 2, ..., as $\hat{\beta}$, the rank estimate that minimizes $D(\beta)$ in (8). Hence $\hat{\beta}^{(k)}$ could be used in W to construct a test. Generally, $\hat{\beta}^{(k)}$ does not converge to $\hat{\beta}$ as k increases. It is probably best to take around 4 or 5 steps and then construct W.

Summary: (1) There are no published studies that show how the level of W behaves for small samples. There is no indication of how large the sample size should be before the asymptotic distribution provides a good approximation. (2) Computation of W requires special programs and cannot be carried out using existing statistical packages. In 1983 the Minitab statistical computing system will contain a command that will produce in its output the rank estimate $\hat{\beta}$ and the test statistic W. There is further discussion of computation in

Section 3. (3) With the exception of the small unpublished simulation by Hettmansperger and McKean (1981) no study of the small sample power of W is available. In the simulation just mentioned the estimate (14) was used after some small sample adjustments. For example τ^* is multiplied by the bias correction $(n/(n-p))^{1/2}$. It was found in the parallelism design that W was often liberal and needed further correction to reduce the probability of a type I error. Its small sample power was comparable to the power of the F, A and D* (in the next subsection) tests.

(iii) Test Based on Reduction due to Fitting the Full and Reduced Models

This method in the rank case is analogous to the F statistic which can be written as the reduction in sum of squares due to fitting the full and reduced models. The aligned rank test and the Wald test are not directly based on the comparison of a reduced and full model. The aligned test is constructed from reduced-model residuals and the Wald test is a quadratic form in the full-model estimates. It might seem most natural to combine estimation with fitting in a robust fashion in order to have a set of strategies parallel to least squares. Then data analytic methods such as plotting have direct counterparts based on ranks.

The test is based on Jaeckel's measure of dispersion (8). Let

$$D^* = \frac{D(\hat{\hat{\beta}}_1) - D(\hat{\beta})}{\hat{\tau}/2}, \tag{16}$$

where $\hat{\hat{\beta}}_1$ and $\hat{\beta}$ are the reduced and full model rank estimates, respectively, and $\hat{\tau}$ is a consistent estimate of τ. Under the technical assumptions of Section 5 McKean and Hettmansperger (1976) show that D* has a limiting chi-square distribution with q degrees of freedom. Hence, the test based on

(16) rejects $H_o:\beta_2=0$ if $D^* \geq \chi^2_\alpha(q)$. Computation of D^* requires special programs for $\hat{\hat{\beta}}_1$, $\hat{\beta}$ and $\hat{\tau}$. The forthcoming Minitab rank-regression command will incorporate this test as part of its output. See Section 3 for further aspects of the computational problems. The test is illustrated on data in Section 4.

Summary: (1) Hettmansperger and McKean (1977) provide a small simulation which indicates that D^* along with τ^*, tuned for small samples, has a significance level close to the nominal level. McKean and Hettmansperger (1978) provide simulation results for the k-step estimate, D^* and τ^*. Again, the test seems to have a stable level. There are no simulation studies of D^* with $\hat{\tau}$. There is no indication of how large the sample size should be before the asymptotic distribution of D^* with $\hat{\tau}$ provides a good approximation. (2) Because of the computational problems involved in computing $\hat{\beta}$ and $\hat{\tau}$ or τ^* (see Section 3) special programs are required to compute D^*. In 1983 the Minitab statistical computing system will produce $\hat{\beta}$, D^* and $\hat{\tau}$ or τ^* in the output of a rank-regression command. (3) In an unpublished simulation study by Hettmansperger and McKean (1981) the test based on D^* with τ^* had power comparable to the F test and the test based on W.

Finally, it should be emphasized that the use of τ^* requires the assumption of symmetry of the error distribution. The estimate $\hat{\tau}$ does not require symmetry. It is not yet known how well $\hat{\tau}$ will work in the asymmetric case and it is not know if $\hat{\tau}$ will be a viable substitute for τ^* in the symmetric case. Consistency of $\hat{\tau}$ has only been established for the Wilcoxon scores. This work has been pursued by Aubuchon (1982).

3. COMPUTATIONS

As was mentioned in Section 2, special programs are necessary to calculate any of the test statistics other than Adichie's (using least squares to fit the reduced model). First of all, a program which minimizes the dispersion, (8), is needed to obtain rank estimates and to evaluate the dispersion for these estimates. Then, in order to use either the Wald quadratic-form test or the drop-in-dispersion test, we need a program to compute an estimate of the scaling functional τ appearing in the denominator. The discussion in this section pertains to procedures generated by the general score functions defined in A9-A11 in Section 5.

An algorithm suggested by J. W. McKean (personal communication) for minimizing the dispersion is perhaps best thought of as a member of the class of iterative schemes known as gradient methods. The increment to the estimate at the K^{th} step is given by a positive step size $t^{(K)}$ times some symmetric, positive definite matrix C times the negative of the gradient:

$$\hat{\beta}^{(K+1)} - \hat{\beta}^{(K)} = t^{(K)} C S(\hat{\beta}^{(K)}) \ . \tag{17}$$

Recall that $-S(\beta)$ is the gradient of the dispersion, (9). Two considerations lead us to set $C = (X_c'X_c)^{-1}$ in (17). First of all, since the asymptotic variance-covariance structure of $\hat{\beta}$ is given by a constant times $(X_c'X_c)^{-1}$, a natural norm for β is $||\beta|| = (\beta'X_c'X_c\beta)^{1/2}$. Results of Ortega and Rheinboldt (1970) show that the direction of steepest descent with respect to this norm is precisely $(X_c'X_c)^{-1}S(\beta^{(K)})$. On the other hand, Jaeckel (1972) shows that the dispersion function may be approximated asymptoticly by a quadratic:

$$D(\beta) \doteq D(\beta_o) - (\beta-\beta_o)'S(\beta_o) + (2\tau)^{-1}(\beta-\beta_o)'X_c'X_c(\beta-\beta_o), \tag{18}$$

where β_o is the vector of true regression parameters. The minimum of this quadratic is attained for

$$\beta = \beta_o + \tau(X_c'X_c)^{-1}S(\beta_o). \tag{19}$$

Thus, if we substitute $\hat{\beta}^{(K)}$, our current estimate, for β_o in (19) we are again led to take a step in the direction $(X_c'X_c)^{-1}S(\hat{\beta}^{(K)})$.

It remains to choose the step size, $t^{(K)}$. We might search for the minimum of $D(\hat{\beta}^{(K+1)})$ as a function of $t^{(K)}$ using any good linear search method--the golden section search or one of the other methods described in Kennedy and Gentle (1980), for example. McKean suggests that this search might be conducted by making use of the asymptotic linearity of the derivative of $D[\hat{\beta}^{(K)} + t(X_c'X_c)^{-1}S(\hat{\beta}^{(K)})]$ with respect to t, given below in (20). (Compare Hettmansperger and McKean (1977).) Specifically, he suggests application of the Illinois version of false position, as discussed by Dowell and Jarratt (1971), to find the approximate root of this derivative:

$$S^*(t) = a'(\hat{\beta}^{(K)})X_c(X_c'X_c)^{-1}X_c'a[\hat{\beta}^{(K)} + t(X_c'X_c)^{-1}S(\hat{\beta}^{(K)})], \tag{20}$$

which is a nondecreasing step function. Whatever linear search method is employed, this approach is equivalent to transforming the linear model by obtaining an orthogonal design matrix and then using the method of steepest descent.

As with any iterative method, it is necessary to specify starting values and convergence criteria. One possibility for $\hat{\beta}^{(0)}$ is the usual least-squares estimate, which is easy to compute and which we would most likely desire for comparative purposes in any case. Another choice would be some more resistant estimate, such as the L_1 estimate, which is, however, more expensive to compute. It is not clear what the trade-offs in computational efficiency would be in making such choices. For a convergence criterion it may be best to focus on relative change in the dispersion, since the value of β which minimizes the dispersion is not generally unique. Criteria which check whether the gradient is (approximately) zero will not be useful, since the gradient is a step function and may step across zero.

If, in (17), we let $C = (X_c'X_c)^{-1}$ as suggested and set $t^{(K)} = \hat{\tau}^{(K)}$, an estimate of τ computed on the residuals at the Kth step, we essentially have an iterative scheme based on the K-step estimates discussed in Section 2ii. While such estimates may be of interest in their own right, early experience of McKean and others indicates that, taken as an algorithm for minimizing the dispersion, this scheme can behave rather poorly for some data sets, failing to converge to, and in fact moving away from, a minimizing point.

We should also mention that Osborne (1981) and others have developed algorithms for minimizing the dispersion using methods of convex analysis.

Although iterative methods are not needed to compute the window estimate of τ, a naive approach will not be very efficient. Schweder (1975) suggests an interesting scheme for computing $\sum_{ij}\sum I\{|r_i - r_j| < h_n/2\}$ but doesn't give details. A time- and space-efficient algorithm based on Schweder's suggestion may be found in Aubuchon (1982).

With the assumption that the error distribution is symmetric, McKean and Hettmansperger (1976) show that a consistent estimate of τ may be obtained by applying a one-sample rank procedure to the uncentered residuals, $r_i = y_i - x_{ic}'\hat{\beta}$, using the one-sample score function corresponding to ϕ: $\phi^+(u) = \phi((u+1)/2)$. If $(\hat{\alpha}_L, \hat{\alpha}_U)$ is the 100 $(1-\alpha)$ % confidence interval obtained for the center of symmetry in this fashion, then $\hat{\tau} = \sqrt{n}\ (\hat{\alpha}_U - \hat{\alpha}_L)/(2Z_{\alpha/2})$ is a consistent estimate of τ. This approach is an extension of the work of Sen (1966) to the linear model.

If Wilcoxon scores are used, there are at least three approaches to obtaining $\hat{\alpha}_L$ and $\hat{\alpha}_U$. If storage space and efficiency are not of critical importance, the $n(n+1)/2$ pairwise (Walsh) averages may be computed. Then $\hat{\alpha}_L$ and $\hat{\alpha}_U$ are the $(c+1)$st and $(n(n+1)/2 - c)$th order statistics from this set, where c is the lower critical point of a two-sided, size-α Wilcoxon signed-rank test. This critical point may be obtained from tables or from a normal approximation. Any fast algorithm for selecting order statistics might then be used to find $\hat{\alpha}_L$ and $\hat{\alpha}_U$; see, for example, Knuth (1973). An approach which is faster and which requires much less storage is based on Johnson and Mizoguchi (1978), with improvements discussed by Johnson and Ryan (1978). These papers actually present the algorithm for the two-sample problem; but simple modifications make it applicable to the present case as well. One advantage of this method is that it still selects exact order statistics from the set of Walsh averages, without computing and storing all of them. A third method, relying on the asymptotic linearity of signed-rank statistics, does not guarantee exact results but is quite fast and space-efficient. The Illinois version of false position is used to find approximate solutions to the equations (21) defining $\hat{\alpha}_L$ and $\hat{\alpha}_U$ in terms of a signed-rank statistic:

$$\begin{aligned} \sqrt{n}\ \bar{V}(\hat{\alpha}_L) &= Z_{\alpha/2} \\ \sqrt{n}\ \bar{V}(\hat{\alpha}_U) &= -Z_{\alpha/2}\ , \end{aligned} \tag{21}$$

where $\bar{V}(\alpha) = n^{-1} \sum_{i=1}^{n} \phi^{+}(R_i^{+}/(n+1))\,\mathrm{sign}(r_i-\alpha)$, R_i^{+} is the rank of $|r_i-\alpha|$ among $|r_1-\alpha|, \ldots , |r_n-\alpha|$ and $Z_{\alpha/2}$ is the upper $\alpha/2$ point of the standard normal distribution. See McKean and Ryan (1977) for the use of this algorithm in the corresponding two-sample problem.

For certain other score functions, for example the scores suggested by Policello and Hettmansperger (1976), $\hat{\alpha}_L$ and $\hat{\alpha}_U$ are order statistics from a well-defined subset of the Walsh averages. In this case, the first two methods discussed above are still applicable. In general, $\hat{\alpha}_L$ and $\hat{\alpha}_U$ are weighted order statistics from the Walsh averages, with weight $a^{+}(j-i+1) - a^{+}(j-i)$ given to $(r_{(i)} + r_{(j)})/2$, where $a^{+}(i) = a^{+}(i/(n+1))$; see Bauer (1972). Thus, if a program for selecting weighted order statistics is available, the first method still works. Otherwise, the third method may be used with any score function.

4. EXAMPLE

In order to illustrate the procedures discussed in this paper, we have applied them to data from an experiment described by Shirley (1981). In this section computations are based on the Wilcoxon scores in (6). Two censored observations are recorded at the censoring point for the purposes of this example. The data are displayed in Table 1. Thirty rats received a treatment intended to delay entry into a chamber. The rats were divided into three groups of ten, a control group and two experimental groups. The experimental groups each received some antidote to the treatment, while the control group received none. The time taken by each rat to enter the chamber was recorded before the treatment and again after the treatment and antidote - if any.

- Table 1 About Here -

We consider the measurement before treatment as a covariate and test for interaction between the grouping factor and the covariate; i.e., we test for equal slopes. The observations are strongly skewed; we applied the natural log transformation to gain some degree of symmetry so that the estimate τ^* in (14) may be applied to the data.

Computations for the aligned rank test, using least squares to fit the reduced model, can be carried out in the SAS statistical computing system (see Helwig and Council (1979)) using the following program:

```
DATA;
    INPUT BEFORE AFTER ANTIDOTE;
    LOG_AFT = LOG (AFTER);
    CARDS;
{data goes here}

PROC GLM;
    CLASS ANTIDOTE;
    MODEL LOG_AFT = ANTIDOTE BEFORE;
    OUTPUT OUT=RESID RESID=RESID;
PROC SORT DATA=RESID;
    BY RESID;
DATA RSCORE;
    SET RESID;
    RSCORE = SQRT(12) * (_N_/31 - .5);
PROC GLM DATA=RSCORE;
    CLASS ANTIDOTE;
    MODEL  RSCORE = ANTIDOTE BEFORE ANTIDOTE*BEFORE;
```

The desired test statistic will be the Type 4 sum of squares for ANTIDOTE*BEFORE in the second GLM output.

The same calculation can be made in the Minitab statistical computing system (see Ryan, Joiner and Ryan (1981)). Some manipulation is necessary to create the design matrix so that the REGRESS command can be used. Indicator variables for the first two groups are put in 'A1' and 'A2'; then two interaction columns, 'INTER1' and 'INTER2', are produced by multiplying each of these by the covariate.

```
NAME C1 = 'BEFORE' C2 = 'AFTER' C3 = 'ANTIDOTE'
NAME C4 = 'LOG.AFT' C5 = 'A1' C6 = 'A2'
NAME C7 = 'A3' C8 = 'INTER1' C9 = 'INTER2'
NAME C10 = 'STD.RES.' C11 = 'FITS' C12 = 'RANKS'
NAME C13 = 'RSCORES' C14 = 'RESIDS'

READ 'BEFORE' 'AFTER' 'ANTIDOTE'

{Data goes here}

LET 'LOG.AFT.' = LOGE('AFTER')
INDICATORS FOR 'ANTIDOTE' IN 'A1' 'A2' 'A3'
LET 'INTER1' = 'A1' * 'BEFORE'
LET 'INTER2' = 'A2' * 'BEFORE'
REGRESS 'LOG.AFT' 3 'A1' 'A2' 'BEFORE' 'STD.RES.'
                     'FITS'
```

```
LET 'RESIDS' = 'LOG.AFT.' - 'FITS'
RANKS OF 'RESIDS' IN 'RANKS'
LET 'RSCORES' = SQRT(12) * ('RANKS'/31 - .5)
REGRESS 'RSCORES' 5 'A1' 'A2' 'BEFORE' 'INTER1' 'INTER2'
```

The test statistic is then the sum of the last two sums of squares in the table labeled "SS Explained By Each Variable When Entered In The Order Given." In general, the columns to be tested should be given last in the REGRESS command used to fit the full model to the rank scores.

The divisor 31 used to calculate the rank scores in the two programs corresponds to the quantity (n+1). Using either program, we have the value .42 for the test statistic. When this is compared to a χ^2 critical point with two degrees of freedom, we fail to reject the hypothesis of equal slopes at any reasonable level. We could now proceed to perform similar tests for the group effect and for the covariate.

A program implementing the algorithms described in Section 3 in Fortran was used to perform the Wald test and the drop-in-dispersion test for the equal slopes hypothesis. Both of the estimates for τ discussed in Section 2ii were employed. The results are presented in detail for comparison with other programs for minimizing the dispersion. In Table 2, the fitting of full and reduced models is summarized. The values in parentheses correspond to the least-squares estimates, which were used as starting values.

- Table 2 About Here -

Seven steps were required to attain covergence of the full-model estimates, while three steps were used for the reduced-model estimates. We report the two estimates of τ, given by $\hat{\tau} = (12\ \hat{\gamma}^2)^{-1}$ for $\hat{\gamma}$ in (12) and by τ^* in (14). The least-squares estimate of σ is also shown.

In Table 3 we present four test statistics, corresponding to use of either the Wald test statistic in (11) or the drop test statistic in (16) combined with either $\hat{\tau}$ or τ^* as an estimate of τ in the denominator. For comparison, we also list the aligned rank test statistic computed above, as well as twice the usual least-squares F statistic. All of these statistics

- Table 3 About Here -

may be compared to the upper α-point of the chi-square distribution with two degrees of freedom. In practice, we would most likely apply some small-sample tuning to τ^* or $\hat{\tau}$. Further, we would divide all of the test statistics except, perhaps, Adichie's by the numerator degrees of freedom q ($q = 2$ for this example) and compare the results to the upper α-point of the F distribution with q and n-p-1 degrees of freedom.

5. EXTENSIONS AND TECHNICAL ASSUMPTIONS

The procedures discussed in Section 2 may be generalized by use of scores other than Wilcoxon scores. The technical assumptions used by different authors to obtain results for these procedures and their generalizations have shown considerable variation. Two papers establishing asymptotic linearity of rank statistics for general linear models, those of Jureckova (1971a) and Kraft and van Eeden (1972), are of central importance for the theory of rank tests and estimates in these models. Other authors frequently rely on the results of these papers and state some subset of the assumptions of one or the other or both. It is not always clear that the subset given in a particular paper is actually sufficient, since details of proofs are often omitted. For our purpose it seems adequate to list for comparison the assumptions made to obtain the linearity results in these two seminal papers. We consider in turn three categories of assumptions: on the error distribution, on the design matrix and on the scores. We will then state the assumptions of the principle papers establishing properties of the procedures we have discussed. Before continuing we should note that different versions of the test statistics discussed here may also be obtained by use of signed-rank scores rather than the rank scores which we have used. See Kraft and van Eeden (1972) and Jureckova (1971b) for asymptotic linearity results in this case.

The two papers, Jureckova (1971a) and Kraft and van Eeden (1972), make the same assumptions about the error distribution.

<u>ASSUMPTION</u>: A6. F has density f with finite Fisher information; that is, f is absolutely continuous and $\int_{-\infty}^{\infty} (f'(y)/f(y))^2 f(y)dy < \infty$.

Beyond the simple assumption (A2) that X_c has full rank, requirements for the design matrix are rather complicated.

<u>ASSUMPTION</u>: A7 (Jureckova):

(a) $n^{-1}X_c{}'X_c \to \Sigma$, a positive definite matrix, (A4);

(b) $X^{(1)} = X^{(1)} + X^{(2)}$, $X^{(1)}$ having nondecreasing columns, $X^{(2)}$ nonincreasing.

This decomposition of X is stated somewhat differently from what appears in Jureckova (1971a), but the two versions amount to the same thing. Such a decomposition is, in fact, always possible; but for $C^{(m)} = X^{(m)} - 1\bar{X}^{(m)}{}'$ $(m = 1,2)$ and for each $j = 1, 2, \ldots, p$, one of the following two conditions must also be satisfied:

(i) the jth column of $C^{(m)}$ is zero for all but a finite number of n; or

(ii) the jth column of $C^{(m)}$ is nonzero for all but a finite number of n, $n^{-1}\sum_{i=1}^{n}(C_{ij}{}^{(m)})^2$ is bounded, and the column satisfies Noether's condition:

$$\lim_{n\to\infty} \frac{\max_i (C_{ij}{}^{(m)})^2}{\sum_{i=1}^{n}(C_{ij}{}^{(m)})^2} = 0 .$$

<u>ASSUMPTION</u>: A8 (Kraft and van Eeden):

(a) $n^{-1}X_c{}'X_c \to \Sigma$, a positive definite matrix, A4;

(b) each column of X_c satisfies Noether's condition:

$$\lim_{n\to\infty} \frac{\max_i (x_{ijc})^2}{\sum_{i=1}^{n}(x_{ijc})^2} = 0 .$$

(c) For any pair of columns $x_c{}^{(j)}$ and $x_c{}^{(k)}$ $(j\neq k)$ of X, there exists $\gamma_{jk}\neq 0$ and N such that $n > N$ implies $x_c{}^{(j)}$ and $x_c{}^{(j)} + \gamma_{jk}x_c{}^{(k)}$ are similarly ordered. Two vectors z and w are similarly ordered if $(z_i-z_j)(w_i-w_j) \geq 0$ for every pair (i,j).

Usually the scores are generated from a function ϕ on (0,1) in some manner.

<u>ASSUMPTION</u>: A9 (Jureckova):

Suppose ϕ is a nonconstant, nondecreasing, square-integrable function on (0,1) or the difference of two such functions. Then we may let the scores be $a(i) = \phi(i/(n+1))$ or $a(i) = E[\phi(U^{(i)})]$, where $U^{(i)}$ is the ith order statistic from a sample of size n from the uniform distribution on (0,1).

<u>ASSUMPTION</u>: A10 (Kraft and van Eeden):

Suppose G is a distribution function with absolutely continuous density g. Suppose further that $\phi_g(u) = -g'(G^{-1}(u))/g(G^{-1}(u))$ is the sum of two monotone, square-integrable functions on (0,1). Then we may let $a(i) = \phi(i/(n+1))$.

<u>ASSUMPTION</u>: A11.

We require that $\int_0^1 \phi(u) = 0$ and $\int_0^1 \phi^2(u)du = 1$. Score functions which are square-integrable may always be standardized in this way without affecting the properties of the resulting tests or estimates. In addition, we define $\tau = [\int_0^1 \phi(u)\phi_f(u)du]^{-1}$; this is the scaling functional appearing in the asymptotic variance-covariance matrix of the rank estimates and in the denominators of the Wald and drop-in-dispersion test statistics.

We now discuss each of the procedures, stating the assumptions required.

<u>Adichie's Aligned Rank Test</u>. The common assumptions A6 about the error distribution are adopted. As Adichie (1978) states his results, the nx1 vector of ones, 1_n, can not be in the column space of the design matrix.

Thus, in particular, the model can not include an intercept. This seems to be a serious shortcoming for practical applications. However, it is possible to deal with the problem. Consider the model: $Y = \alpha 1_n + X\beta + e = \alpha^* 1_n + X_c\beta + e$. Since X_c is centered, 1_n is not in its column space. Thus, if we let $Y^* = Y - \alpha^* 1_n$, Adichie's results would apply to the model $Y^* = X_c\beta + e$, allowing us to test hypotheses about β. The fact that Y^* is unobservable since α^* is unknown doesn't matter. Adichie's statistic computed on Y^* is exactly the same as that computed on Y, which we do observe. For the rest, Adichie states that either the conditions A8 on the design of Kraft and van Eeden (1972), or those of Jureckova (1971a) A7 will suffice. Adichie generates the scores as $a(i) = \phi(i/(n+1))$, where ϕ satisfies the requirements A9 of Jureckova. Thus, as we have stated Adichie's procedure, assumptions A2, A3, A6, A9, A11 and either A7 or A8 are needed.

Wald Quadratic-Form Test. This test relies only on the asymptotic distribution of the rank estimates used. Thus either the conditions of Kraft and van Eeden (1972) or of Jureckova (1971a) will be sufficient; that is, either A2, A6, A8, A10 and A11 or A2, A6, A7, A9 and A11.

Drop-in-Dispersion Test. McKean and Hettmansperger (1976) establish the asymptotic properties of this test statistic taking Jureckova's (1971a) assumptions for a foundation, with the additional requirement that $\Sigma a(i)=0$. It would seem that the results could also be obtained under the Kraft and van Eeden (1972) conditions, although this has not been carried out. Thus assumptions A2, A6, A7, A9 and A11 are needed.

These last two test procedures require a consistent estimate of the scaling functional τ. If the estimate suggested by McKean and Hettmansperger (1976) is used, it must be assumed that the error distribution is symmetric.

In extending the window estimate of τ to the linear model, Aubuchon (1982) required the density to have a bounded second derivative. This extension is applicable only for Wilcoxon scores.

Table 1. Times Taken for Rats to Enter Cages

Group 1		Group 2		Group 3	
Before treatment	After treatment	Before treatment	After treatment	Before treatment	After treatment
1.8	79.1	1.6	10.2	1.3	14.8
1.3	47.6	0.9	3.4	2.3	30.7
1.8	64.4	1.5	9.9	0.9	7.7
1.1	68.7	1.6	3.7	1.9	63.9
2.5	180.0+	2.6	39.3	1.2	3.5
1.0	27.3	1.4	34.0	1.3	10.0
1.1	56.4	2.0	40.7	1.2	6.9
2.3	163.3	0.9	10.5	2.4	22.5
2.4	180.0+	1.6	0.8	1.4	11.4
2.8	132.4	1.2	4.9	0.8	3.3

+Censored observations.

Table 2. Fitting Full and Reduced Models[a]

	Full	Reduced
Dispersion	17.942 (18.126)	18.306 (18.372)
$\hat{\alpha}$ = Intercept	.664 (.465)	.836 (.874)
$\hat{\beta}_1$ = Antidote 1 - Antidote 3	2.20 (2.43)	1.60 (1.61)
$\hat{\beta}_2$ = Antidote 2 - Antidote 3	-3.90 (-.020)	-.235 (-.342)
$\hat{\beta}_3$ = Before	1.20 (1.35)	1.10 (1.08)
$\hat{\beta}_4$ = Slope for Antidote 1 - Slope for Antidote 3	-.323 (-.502)	---
$\hat{\beta}_5$ = Slope for Antidote 2 - Slope for Antidote 3	.101 (-.221)	---

[a]Tabled numbers correspond to procedures based on Wilcoxon scores; numbers in parentheses correspond to least-squares.

NOTE: $\hat{\tau} = .5309$ $\tau^* = .5215$ $\hat{\sigma} = .7884$

Table 3. Tests for Equal Slopes

Estimate of τ	Test statistic	
	W	D*
$\hat{\tau}$	1.12	1.38
τ^*	1.15	1.40

NOTE: A = .42 2F = .67

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The current status, including small-sample behavior and ease of computation, of rank-based estimates and tests in the general linear model is reviewed. For the important special case of Wilcoxon scores, details of application of various procedures are discussed. The three different testing methods considered may each be motivated by connecting it to one of three forms of the usual least-squares F statistic. Possible algorithms for computation of rank-based esimates and tests are presented. Each procedure is applied to

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an example using data. Finally, the technical assumptions made to obtain large-sample properties of these procedures, including the general-scores case, are outlined and discussed.

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